

Correction : Réviser les factorisations

Exercice 1

$$A = 4x^3 - 5x^2 = x^2(4x - 5)$$

$$B = x^5 + \frac{1}{2}x^3 = x^3\left(x^2 + \frac{1}{2}\right)$$

$$C = (x+1)^2 + x + 1 = (x+1)(x+1+1) = (x+1)(x+2)$$

$$D = 2x + \frac{x}{4} + x(x-2) = x\left(2 + \frac{1}{4} + x - 2\right) = x\left(\frac{1}{4} + x\right)$$

$$E = (3x-7)^2 + (x-2)(7-3x)$$

$$E = (3x-7)^2 + (x-2)(-(-7+3x))$$

$$E = (3x-7)^2 - (x-2)(-7+3x)$$

$$E = (3x-7)(3x-7-x+2)$$

$$E = (3x-7)(2x-5)$$

Exercice 2

$$F = x^2 - 8x + 16 = (x-4)^2$$

$$G = 49 - x^2 = (7-x)(7+x)$$

$$H = -25 + x^2 = x^2 - 25 = (x-5)(x+5)$$

$$I = 8x + 4 + 4x^2 = (2x+2)^2$$

$$J = (3x+2)^2 - (x+1)^2 = (3x+2-x-1)(3x+2+x+1) = (2x+1)(4x+3)$$

Exercice 3

$$\begin{aligned} K &= (2x-3) + (5x+1)(3-2x) \\ &= (2x-3) + (5x+1)(-(-3+2x)) \\ &= (2x-3) - (5x+1)(-3+2x) \\ &= (2x-3)(1-5x-1) \\ &= (2x-3)(-5x) \\ &= 5x(3-2x) \end{aligned}$$

$$\begin{aligned} L &= 9(x-3)^2 - (4x+3)^2 \\ &= 3^2(x-3)^2 - (4x+3)^2 \\ &= (3(x-3))^2 - (4x+3)^2 \\ &= (3(x-3) - 4x-3)(3(x-3) + 4x+3) \\ &= (-x-12)(7x-6) \\ &= -(x+12)(7x-6) \end{aligned}$$

$$\begin{aligned} M &= -(1+3x)^2 + 4x^2 \\ &= 4x^2 - (1+3x)^2 \\ &= (2x-1-3x)(2x+1+3x) \\ &= (-x-1)(5x+1) \\ &= -(x+1)(5x+1) \end{aligned}$$

$$N = (1-3x)^2 + 3(1-2x) = 1 - 6x + 9x^2 + 3 - 6x = 9x^2 - 12x + 4 = (3x-2)^2$$

$$O = \left(\frac{x-3}{2}\right)^2 - \frac{x^2}{4} = \left(\frac{x-3}{2}\right)^2 - \left(\frac{x}{2}\right)^2 = \left(\frac{x-3-x}{2}\right)\left(\frac{x-3+x}{2}\right) = -\frac{3}{2} \times \frac{2x-3}{2} = \frac{3(3-2x)}{4}$$

Exercise 4

$$1) x^2 = 25$$

$$\Leftrightarrow x^2 - 25 = 0$$

$$\Leftrightarrow (x-5)(x+5) = 0$$

$$\Leftrightarrow x = 5 \text{ ou } x = -5$$

$$2) \left(x - \frac{1}{5}\right)^2 = 9$$

$$\Leftrightarrow \left(x - \frac{1}{5}\right)^2 - 9 = 0$$

$$\Leftrightarrow \left(x - \frac{1}{5} - 3\right)\left(x - \frac{1}{5} + 3\right) = 0$$

$$\Leftrightarrow \left(x - \frac{1}{5} - \frac{15}{5}\right)\left(x - \frac{1}{5} + \frac{15}{5}\right) = 0$$

$$\Leftrightarrow \left(x - \frac{16}{5}\right)\left(x + \frac{14}{5}\right) = 0$$

$$\Leftrightarrow x = \frac{16}{5} \text{ ou } x = -\frac{14}{5}$$

$$3) (2x-5)(x-1) = 6x-15$$

$$\Leftrightarrow (2x-5)(x-1) - 6x + 15 = 0$$

$$\Leftrightarrow (2x-5)(x-1) - (6x-15) = 0$$

$$\Leftrightarrow (2x-5)(x-1) - (2x-5) \times 3 = 0$$

$$\Leftrightarrow (2x-5)(x-1-3) = 0$$

$$\Leftrightarrow (2x-5)(x-4) = 0$$

$$\Leftrightarrow 2x-5=0 \text{ ou } x-4=0$$

$$\Leftrightarrow x = \frac{5}{2} \text{ ou } x = 4$$

$$4) \left(x + \frac{1}{2}\right)(-x+1) = -1-2x$$

$$\Leftrightarrow \left(x + \frac{1}{2}\right)(-x+1) + 1 + 2x = 0$$

$$\Leftrightarrow \left(x + \frac{1}{2}\right)(-x+1) + \left(\frac{1}{2} + x\right) \times 2 = 0$$

$$\Leftrightarrow \left(x + \frac{1}{2}\right)(-x+1+2) = 0$$

$$\Leftrightarrow \left(x + \frac{1}{2}\right)(-x+3) = 0$$

$$\Leftrightarrow x + \frac{1}{2} = 0 \text{ ou } -x+3=0$$

$$\Leftrightarrow x = -\frac{1}{2} \text{ ou } x = 3$$

$$5) (2x+1)^2 - (x-2)^2 = 0$$

$$\Leftrightarrow (2x+1+x-2)(2x+1-x+2) = 0$$

$$\Leftrightarrow (3x-1)(x+3) = 0$$

$$\Leftrightarrow x = \frac{1}{3} \text{ ou } x = -3$$

Exercise 5

$$1) -\left(\frac{x-1}{2}\right)^2 + 9x^2 > 0$$

$$\Leftrightarrow (3x)^2 - \left(\frac{x-1}{2}\right)^2 > 0$$

$$\Leftrightarrow \left(3x - \frac{x-1}{2}\right)\left(3x + \frac{x-1}{2}\right) > 0$$

$$\Leftrightarrow \left(\frac{6x-x+1}{2}\right)\left(\frac{6x+x-1}{2}\right) > 0$$

$$\Leftrightarrow \left(\frac{5x+1}{2}\right)\left(\frac{7x-1}{2}\right) > 0$$

$$\Leftrightarrow (5x+1)(7x-1) > 0$$

x	$-\infty$	$-\frac{1}{5}$	$\frac{1}{7}$	$+\infty$		
$5x+1$		-	0	+	+	
$7x-1$		-	0	-	+	
$(5x+1)(7x-1)$		+	0	-	0	+

$$S =]-\infty; -\frac{1}{5}[\cup]\frac{1}{7}; +\infty[$$

$$\begin{aligned}
2) & \left(\frac{3}{2}x-1\right)(x+1) + \frac{1}{2}x \leq \frac{1}{3} \\
& \Leftrightarrow \left(\frac{3}{2}x-1\right)(x+1) + \frac{1}{2}x - \frac{1}{3} \leq 0 \\
& \Leftrightarrow \left(\frac{1}{2}x - \frac{1}{3}\right) \times 3 \times (x+1) + \frac{1}{2}x - \frac{1}{3} \leq 0 \\
& \Leftrightarrow \left(\frac{1}{2}x - \frac{1}{3}\right)(3 \times (x+1) + 1) \leq 0 \\
& \Leftrightarrow \left(\frac{1}{2}x - \frac{1}{3}\right)(3x+4) \leq 0
\end{aligned}$$

x	$-\infty$	$-\frac{4}{3}$	$\frac{2}{3}$	$+\infty$		
$\frac{1}{2}x - \frac{1}{3}$		-	-	0	+	
$3x+4$		-	0	+	+	
$\left(\frac{1}{2}x - \frac{1}{3}\right)(3x+4)$		+	0	-	0	+

$$S = \left[-\frac{4}{3}; \frac{2}{3}\right]$$

$$\begin{aligned}
3) & 2x(x-1) < 3x-3 \\
& \Leftrightarrow 2x(x-1) - 3x + 3 < 0 \\
& \Leftrightarrow 2x(x-1) - (3x-3) < 0 \\
& \Leftrightarrow 2x(x-1) - 3(x-1) < 0 \\
& \Leftrightarrow (x-1)(2x-3) < 0
\end{aligned}$$

x	$-\infty$	1	$\frac{3}{2}$	$+\infty$		
$x-1$		-	0	+	+	
$2x-3$		-	-	0	+	
$(x-1)(2x-3)$		+	0	-	0	+

$$S =]1; \frac{3}{2}[$$

$$\begin{aligned}
4) & 5x^2 \geq 15x \\
& \Leftrightarrow 5x^2 - 15x \geq 0 \\
& \Leftrightarrow 5x(x-3) \geq 0
\end{aligned}$$

x	$-\infty$	0	3	$+\infty$
$5x$		0	$+$	$+$
$x-3$		$-$	0	$+$
$5x(x-3)$		$+$	0	$+$

$$S =]-\infty; 0] \cup [3; +\infty[$$

$$\begin{aligned}
5) \quad & \frac{(x-2)^2}{4} \leq \frac{x^2}{9} \\
\Leftrightarrow & \frac{(x-2)^2}{4} - \frac{x^2}{9} \leq 0 \\
\Leftrightarrow & \left(\frac{x-2}{2}\right)^2 - \left(\frac{x}{3}\right)^2 \leq 0 \\
\Leftrightarrow & \left(\frac{x-2}{2} - \frac{x}{3}\right) \left(\frac{x-2}{2} + \frac{x}{3}\right) \leq 0 \\
\Leftrightarrow & \left(\frac{3x-6-2x}{6}\right) \left(\frac{3x-6+2x}{6}\right) \leq 0 \\
\Leftrightarrow & \left(\frac{x-6}{6}\right) \left(\frac{5x-6}{6}\right) \leq 0 \\
\Leftrightarrow & (x-6)(5x-6) \leq 0
\end{aligned}$$

x	$-\infty$	$\frac{6}{5}$	6	$+\infty$
$5x-6$		0	$+$	$+$
$x-6$		$-$	0	$+$
$(x-6)(5x-6)$		$+$	0	$+$

$$S = \left[\frac{6}{5}; 6\right]$$